

Mathematical Appendix

Most of the methodology I used followed closely to David Robinson's, replacing at-bats and hits with shots and goals respectively.

Initial Bayesian Estimates

The initial Bayesian model is estimated using a Beta prior distribution with the following distribution of conversion percentage c_i (values determined through empirical distribution fitting):

$$c_i \sim \text{beta}(\alpha_0, \beta_0)$$

$$\alpha_0 = 3.26$$

$$\beta_0 = 32.5$$

This means the prior point estimate for each player is just the mean:

$$\mu_0 = \alpha_0 / (\alpha_0 + \beta_0) \approx 0.0912$$

And the conversion percentage point estimates are derived as such:

$$\hat{c}_i = (\text{goals}_i + \alpha_0) / (\text{shots}_i + \alpha_0 + \beta_0)$$

Beta-Binomial Regression Estimates

With the Beta-Binomial regression we assume the following two linked distributions:

$$c_i \sim \text{beta}(\alpha_0, \beta_0)$$

$$\text{goals}_i \sim \text{binomial}(\text{shots}_i, c_i)$$

We then parameterise using the following approach

$$\sigma_0 = 1 / (\alpha_0 + \beta_0)$$

$$\mu_0 = \alpha_0 / (\alpha_0 + \beta_0)$$

$$c_i \sim \text{beta}(\mu_0 / \sigma_0, (1 - \mu_0) / \sigma_0)$$

Now we can define individual priors for each player (the coefficients are estimated using a logistic regression):

$$\mu_i = \mu_0 + \mu_{\text{shots}} \log(\text{shots}_i)$$

And individualised alphas and betas:

$$\alpha_{0,i} = \mu_i / \sigma_0$$

$$\beta_{0,i} = (1 - \mu_i) / \sigma_0$$

And now the prior distributions for each player are individualised as well and we can use these priors to estimate the conversion percentages in the same Bayesian way as above:

$$c_i \sim \text{beta}(\alpha_{0,i}, \beta_{0,i})$$

$$\text{goals}_i \sim \text{binomial}(\text{shots}_i, c_i)$$